



International Education Agency of PNG

# Mathematics Curriculum

Revised Edition – 2006

## Overview

Mathematics is an integral part of our existence. It is a powerful form of communication which enables us to represent, to interpret, to explain and to predict.

The study of mathematics involves a search for patterns and relationships through which we are able to explore, explain and interpret the world around us. The focus on problem solving and investigation, which has become a significant element of mathematics learning in recent years, ensures that these possibilities are realised.

We all use at least a fundamental knowledge of numbers, measurement, spatial relationships and statistics in our daily lives. At the same time, more sophisticated understandings in these areas form the basis for a myriad of vital activities in science, the humanities and the arts. Beyond this specific value, the study of mathematics provides opportunities to develop logical reasoning and provides a powerful means of communication.

Mathematics is exciting, challenging and satisfying. It holds an inherent interest for young children, and this can be maintained through effective teaching which emphasises practical relevance.

### Curriculum Outcomes

The mathematics curriculum encompasses many skills which help students function effectively, both personally and vocationally. Mathematics where possible needs to be developed in contexts which are meaningful to real life situations. The mathematics curriculum strives to develop in students a range of skills and understandings.

In particular, teaching and learning in mathematics is directed towards enabling all students to:

- use a variety of strategies to apply understandings of operations, relations and patterns with numbers, and use equations and functions;
- explore and communicate spatial relations and geometry dealing with locations, shapes, patterns, movements and transformations;
- measure;
- collect, communicate, analyse and interpret data;
- explore, analyse and creatively solve problems, apply mathematics and use appropriate mathematical language;
- use estimation and approximation and check the reasonableness of results;
- develop positive attitudes towards exploring, learning and using mathematics.

### Mathematics and the IEA Key Outcomes

IEA schools will assist all children to:

... *Be Self-Directing*

Mathematics is a tool for life which improves the capacity to analyse and deal with every day situations, and in this way empowers learners to pursue their own interests. At the same time, mathematics education involves providing students with a chance to investigate their world using the tools of mathematics.

### *... Communicate Effectively*

The language of mathematics is part of everyday life and communications. Every time we conduct business, do our shopping, or enjoy sports we need diverse knowledge of number, shape and measurement to be able to communicate effectively. At the same time, the huge recent growth in the availability of information through radio, television and the newspaper has led to a need for basic skills in statistics and the ability to consider how reasonable the information we are receiving really is.

### *... Behave Ethically*

A knowledge and sense of mathematics allows us to consider the ethics of situations more accurately. Often ethical arguments are supported with numerical and statistical evidence. Interpreted incorrectly these can lead to serious errors of judgement. Through learning in mathematics students become more skilled at testing assertions and hypotheses. They begin to realise the power of mathematics in supporting arguments and have a chance to consider the ethical consequences of the misuse of this power.

### *... Work Collaboratively*

Many of the activities which involve working with others in the real world require some skills in mathematics. For example, in most cases we make personal and professional financial and budgeting decisions in consultation with others. Additionally, mathematical investigations and problem solving often require the combined efforts of a group, giving an opportunity for this skill to develop.

### *... Analyse and Solve Problems*

Problem solving is at the very core of mathematics. Virtually all mathematical endeavour is concerned with applying learning to real world situations and hence to solving problems. While mathematics is a wonderful tool for the analysis and solution of problems, the skills of problem solving learnt in mathematics can be applied in a wide range of situations.

## **Mathematics in the PNG Context**

Teachers of Mathematics must show an awareness of and respect for the traditional cultural heritage of their students whether from Papua New Guinea or overseas. There needs to be an awareness that some language does not have transferable concepts. Children need to be allowed to compare and discuss mathematical words that are different in their language.

At the same time, the very diversity of the nature of indigenous mathematics offers both a challenge to teachers, and a rich variety from which they may select materials to enhance and enrich their mathematics program. This could be addressed with a possible focus on:

Involving children and the school community in project based exploration of mathematical concepts and practices. This could involve such things as

- Comparing counting systems, bilum making, mapping, basketry, or mat making
- Using natural materials in counting, space and other mathematical topics
- Exploring cyclic time as well as calendar time

- Investigating measurement based upon natural events and processes in contrast to lineal measurement
- Exploring patterns and symbols in PNG arts and crafts
- Playing traditional PNG games
- Examining different PNG mathematical systems and compare these to Hindu-Arabic and Greco-Roman systems.

A PNG Perspective on mathematics will be apparent when teachers, children and the community:

- Think about mathematics as another form of cultural knowledge
- Utilise and value the knowledge of traditional practices of the children and the community of their school
- Actively think about how to go about educating children to consider mathematics as part of their culture
- Use ideas from many cultural backgrounds around PNG to enrich the understanding and teaching of mathematics
- Make connections with the local culture and understand more about the kinds of knowledge that children bring into the classroom
- Are aware of cultural issues that may be sensitive. Children may be reluctant to respond to questions because of cultural restrictions
- Are aware that some traditional activities are gender specific and the mathematical language reflects this
- Are encouraged to discuss and compare maths terminology and concepts in different PNG languages and cultures
- Encourage children to find differences, and to explore and value these differences
- See the reason and importance of using their knowledge so that they are not alienated from their cultural knowledge and values.

## The Mathematics Curriculum Strands

The Mathematics curriculum is organised around four strands.

### **Number**

A sound understanding of numbers is the basis of mathematics. Numbers are the fundamental elements of the language of mathematics. In this strand students are provided with opportunities to explore the nature of numbers and the ways in which they are represented. Skills are developed within the four basic operations, and students learn to manipulate numbers in a variety of ways including algebraically. It encompasses the development of number sense and confidence and competence in using mental, written and calculator techniques for solving problems. Formal written algorithms are introduced after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two-digit and three-digit numbers.

The Number strand has four aspects:

- *Representing Numbers*, - focusing on the representation of numbers including place value, powers, negative signs, square roots and algebraic representations
- *Addition & Subtraction*
- *Multiplication & Division*
- *Fractions and Decimals* - the nature of numbers which are not whole, including percentages and ratios.

## Statistics

In a democracy citizens are provided with a wide range of information which they are expected to use in making decisions about their own lives, at home, in the workplace and in the broader community. Much of this information is presented as data in mathematical forms, and the analysis of data of this type is called statistics. The strand is concerned with the concept of chance and the analysis of data as a means of determining the credibility of a statement or an argument.

Like the other strands, statistics provides widespread opportunities for integration with other curriculum areas. The data provided in the study of science or the social sciences, should provide a rich source for application of new skills learnt within this strand.

## Measurement

The application of understandings about numbers to the measurement of objects or time is a common daily use of mathematics. It relates, for example, to cooking, building, sewing and travelling. At higher levels, measurement is a vital component in engineering and science.

Measurement enables the identification and quantification of time and the attributes of objects so that they can be compared and ordered. Students need to be able to select and use appropriate units and measuring tools, and to estimate and calculate measurements given particular information.

The Measurement strand has seven aspects:

- length, perimeter and circumference
- area
- volume and capacity
- mass
- temperature
- time
- radial measures

## Space

Like the other content strands, spatial concepts are relevant to our daily lives. We use these concepts in organising our living and working areas and in solving problems of design. It involves representation of shape, size, pattern, position and movement of objects in the three-dimensional world, or in the mind of the learner. Students learn to recognise, visualise and draw

shapes and describe the features and properties of three-dimensional objects and two-dimensional shapes in static and dynamic situations. It includes the geometry of transformation and symmetry.

## **Working Mathematically**

Learning and using mathematics involves both an understanding of certain concepts and an ability to apply appropriate processes. The notion of *working mathematically* is a broad approach to mathematics learning which should encompass all strands and be a key focus of all mathematical activity

Working mathematically refers to a broad range of learning outcomes which deal with communicating using mathematical language, reflecting, problem investigation and problem solving strategies, estimation, checks of reasonableness, the social and commercial use of mathematics and logical thinking.

A focus on working mathematically also provides opportunities for learning about the historical and cultural aspects of mathematics, and for exploring the ways in which mathematics affects our lives. Working Mathematically is not a separate curriculum strand which exists in isolation. It is a set of skills and understandings which will be developed within the context provided by each of the four curriculum strands.

Working Mathematically specifically refers to a set of clearly defined skills and processes. These are:

### **1 Asking Questions**

Students must be encouraged to ask questions to analyse and interpret mathematical situations and their personal mathematical experiences. They will often find that they have the skills to answer some of these questions for themselves. Furthermore, the decision to introduce new mathematical concepts can often be timed to meet a need which arises directly from a student's questions.

### **2 Applying Strategies**

While students may learn to use a variety of strategies in a decontextualised way they must be given numerous opportunities to develop, select and use a range of mental and written strategies, including the selection and use of appropriate technology, to explore and solve problems.

### **3 Communicating Ideas**

Mathematics has its own oral and written language. Students need support to develop and use this appropriate language and representations to formulate and express mathematical ideas

### **4 Thinking**

Mathematics provides ongoing opportunities for students to develop broad thinking skills which can be applied across the curriculum. Through mathematics students learn to use a variety of processes to explore relationships, check solutions and give reasons to support their conclusions.

### **5 Reflecting**

While involved in mathematical learning students should be helped to make connections with, and generalisations about, existing knowledge and understandings by reflecting on their experiences, critical understandings and how they learnt.

## **6 Estimating**

The ability to estimate in all areas of mathematics is vital. Before they use a technique to determine an exact result to any calculation they should be encouraged to develop the capacity to use familiar numbers or units to evaluate, guess, compare, assess, judge, approximate, predict the size, number of items or attributes in mathematical situations. Similarly, the skill of estimation allows students to consider the reasonableness of the results of their formal calculations.

## **7 Engaging with the Social and Commercial Applications of Mathematics**

Mathematics is used daily in our general lives. Students must be made aware of these applications and encouraged to apply their learning appropriately. Examples include the manipulation of money, the consideration of data presented in the media, and problems involving the effective use of space.

## Number

LEVEL ONE	LEVEL TWO	LEVEL THREE	LEVEL FOUR	LEVEL FIVE
<b>Level Outcomes</b>				
1.01. Use, describe and represent whole numbers to two places	2.01 Use, describe and represent whole numbers to at least three places	3.01 Use, describe and represent whole numbers to at least four places	4.01 Use, describe and represent whole numbers to at least six places	5.01 Use and apply place value to represent very large and very small numbers and directed numbers
<b>Contributing Outcomes</b>				
<ul style="list-style-type: none"> <li>▪ Recognise numbers in the range 0 to 20 and beyond;</li> <li>▪ Represent numbers in the range 0 to 20 and beyond using objects, words, pictures and then symbols;</li> <li>▪ Count and order numbers in the range 0 to 20 and beyond in correct forward and backward sequence.</li> <li>▪ Develops concept of ten as ten ones or one ten</li> <li>▪ Recognise, describe, create and continue repeating number patterns that increase or decrease</li> </ul>	<ul style="list-style-type: none"> <li>▪ Count, order, read, group and represent two- and three-digit numbers using objects, words, pictures and then symbols;</li> <li>▪ Count forward and backwards in 100's, 10's and 1's on and off century and decade.</li> <li>▪ Generate, describe and record simple number patterns using a variety of strategies</li> </ul>	<ul style="list-style-type: none"> <li>▪ Count, order, read and record numbers up to four digits using objects, words, pictures and then symbols;</li> <li>▪ Count forward and backwards in 1000's, 100's, 10's and 1's on and off millennium, century and decade.</li> <li>▪ Generate, describe and record number patterns using a variety of strategies;</li> </ul>	<ul style="list-style-type: none"> <li>▪ Count, Order, read and write numbers of any size using objects, words, pictures and symbols;</li> <li>▪ Record, analyse and describe number patterns that involve one operation using tables and words.</li> <li>▪ Demonstrate sufficient mathematical understandings for most day to day activities</li> </ul>	<ul style="list-style-type: none"> <li>▪ Read and write very large and small numbers;</li> <li>▪ Use very large and small numbers in calculations with formal algorithms.</li> <li>▪ Read and write one and two digit directed numbers</li> </ul>
<b>Level Outcomes</b>				
1.02 Add and subtract one and two digit numbers using a variety of strategies	2.02 Add and subtract three digit numbers using a variety of strategies	3.02 Add and subtract four digit numbers using a variety of strategies	4.02 Add and subtract six digit numbers using a variety of strategies	5.02 Perform operations with numbers to one million and understand decimals to one millionth.
<b>Contributing Outcomes</b>				
<ul style="list-style-type: none"> <li>▪ Use concept of 10's and ones as a countable units</li> <li>▪ Use and describe a variety of mental strategies for basic addition and subtraction using objects, words and pictures in everyday situations;</li> <li>▪ Use a part or whole knowledge of numbers to 10 and 20 to add and subtract</li> <li>▪ Represent simple calculations with diagrams and objects;</li> <li>• Record simple calculations using informal methods (e.g. using words and pictures )</li> </ul>	<ul style="list-style-type: none"> <li>▪ Use a part or whole knowledge of numbers to 100 to add and subtract;</li> <li>▪ Use concept of 100's, 10's and ones as a countable units in a variety of mental strategies to add and subtract;</li> <li>▪ Use objects, words, pictures, symbols and basic formal recording methods for addition and subtraction involving one- and two-digit numbers</li> <li>▪ Create, represent and continue a variety of simple number patterns, supplying missing elements in a pattern;</li> </ul>	<ul style="list-style-type: none"> <li>▪ Use concept of grouping in 100's and 1000's to add and subtract;</li> <li>▪ Use a part or whole knowledge of numbers to 1000 to add and subtract</li> <li>▪ Add and subtract two-, three- and four-digit numbers using a variety of mental and written strategies</li> <li>▪ Complete addition and subtraction number sentences by calculating missing values.</li> <li>▪ Generate, describe, complete and record number patterns using a variety of strategies.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Construct, verify and complete number sentences involving the addition and subtraction operations with a variety of numbers;</li> <li>▪ Add and subtract counting numbers of any size using appropriate strategies;</li> <li>▪ Apply formal algorithms to addition and subtraction tasks.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Recognises the properties of special groups of whole numbers and apply a range of strategies to aid computation;</li> <li>▪ Compares, orders and calculates with integers.</li> <li>▪ Add and subtract one and two digit directed numbers on a number line</li> </ul>

Level Outcomes				
1.03 Create and count equal groups in a given collection.	2.03 Use a range of mental strategies to solve multiplication and division tasks;	3.03 Use mental and written strategies for multiplication and division with two and three digit numbers	4.03 Select and apply appropriate strategies to solve problems involving multiplication and division	5.03 Explain the relationships between numbers using simple algebra and coordinates
Contributing Outcomes				
<ul style="list-style-type: none"> <li>▪ Share collections of objects into equal groups</li> <li>▪ Identify and name equal groups according to the number of objects in the group</li> <li>▪ Use equal groups as countable units to find <i>how many</i> equal groups are in a collection (e.g. rhythmic and skip counting strategies)</li> <li>▪ Describe counting in groups using objects, words, pictures and everyday language.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Use objects, words, pictures, symbols and informal recording methods to represent simple multiplication and division facts;</li> <li>▪ Use a range of mental strategies and concrete materials to coordinate counting of equal groups of visible and concealed objects (e.g. use visual markers to coordinate counting in groups);</li> <li>▪ Apply simple basic multiplication and division facts (e.g. for 1, 2, 4, 5 and 10) to a range of multiplication and division tasks (e.g. dividing whole objects and collections into equal shares; finding how many of a group are in a given collection)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Apply basic multiplication and division facts (e.g. for 3, 6, 9, and 12) and understanding of place value to a range of multiplication and division tasks (e.g. dividing whole objects and collections into equal shares; finding how many of a group are in a given collection).</li> <li>▪ Represent and solve multiplication and division calculations using basic formal algorithms.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Select and apply a appropriate formal algorithms to a wide range of problems requiring mathematical calculations</li> <li>▪ Represent multiplication and division tasks as formal algorithms.</li> <li>▪ Construct, verify and complete number sentences involving the multiplication and division with a variety of numbers.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Describe the relationships between numbers and number patterns using words and simple algebra and coordinates in a variety of ways;</li> <li>▪ Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions;</li> <li>▪ Uses algebraic techniques to solve linear equations and simple inequalities.</li> <li>▪ Multiply and divide one and two digit directed numbers on a number line</li> </ul>

Level Outcomes				
1.04 Divide whole objects into parts and identify and describe equal sized parts using fractional names	2.04 Determine the relationship between the number of fractional parts of a whole or set of objects and the size and names of these fractional parts	3.04 Represent compare and order fractions and decimal numbers using concrete materials, words and standard notation.	4.04 Represent, compare, order and perform calculations with fractions, decimals, ratios and percentages using concrete materials, words and standard notation..	5.04 Use a variety of mental and written strategies to solve problems involving all four operations with fractions, decimals, ratios and percentages.
Contributing Outcomes				
<ul style="list-style-type: none"> <li>Form equal groups, share and count collections of discrete objects using equal groups as countable units (e.g. share one packet of 10 lollies between 2 people).</li> <li>Share continuous objects into equal parts (e.g. share one cake equally between two people)</li> <li>Describe halves encountered in everyday contexts, as two equal parts of a continuous object;</li> </ul>	<ul style="list-style-type: none"> <li>Explore, create visual models and describe fractional parts (halves and quarters etc) of objects and collections occurring in everyday situations (e.g. share two pizzas equally among four people; share a packet of 12 lollies among four people)</li> <li>Regroup fractional parts into wholes using concrete materials (e.g. combine nine quarters to form two wholes and one quarter)</li> <li>Compare fractions using concrete materials without using standard fraction notation. (e.g. use fraction pieces to show that three quarters is bigger than one half, but smaller than one whole)</li> </ul>	<ul style="list-style-type: none"> <li>Create concrete models, compare, order and represent commonly used and equivalent fractions (including thirds, fifths and tenths) by considering the number of fractional parts. (e.g. share three pizzas equally among four people; share two packets of 6 lollies among three people)</li> <li>Demonstrate and explain the relationship between equivalent fractions using concrete materials</li> <li>Represent fractions using concrete models, words and standard fraction notation, and explain the meaning of <i>denominator</i> as the number of fractional parts of a whole or set, and the <i>numerator</i> as the number of fractional parts being considered</li> <li>Create concrete models, compare, order and represent <i>decimal numbers</i> to tenths using a variety of ways (concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings).</li> <li>Add and subtract decimals to one decimal place.</li> <li>Create concrete models, compare, order, represent and interpret everyday percentages;</li> </ul>	<ul style="list-style-type: none"> <li>Represent, compare and order fractional amounts with like and unlike denominators including proper and improper fractions and mixed numbers in a variety of ways and standard fractional notation.</li> <li>Demonstrate and explain the concept of equivalent fractions using concrete materials and drawings.</li> <li>Round decimal numbers to nearest tenth in problems arising from real life situations.</li> <li>Create concrete models, compare, order and represent decimal numbers to thousandths using a variety of ways (concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings)</li> <li>Estimate quantities using benchmarks of 10%, 25%, 50%, 75%, and 100% (e.g. the container is 75% full; approx. 50% of students bus to school)</li> <li>Compare, order and calculate with decimals, simple fractions and percentages, ratios and rates.</li> </ul>	<ul style="list-style-type: none"> <li>Demonstrate a single number sense of fractions;</li> <li>Order fractions by using multiplicative structure to create equivalences and estimate location;</li> <li>Add, subtract, multiply and divide whole numbers by simple fractions and by decimal numbers to hundredths using concrete materials (e.g. divide 3 by <math>\frac{1}{2}</math> using fraction strips; divide 4 by 0.8 using base ten materials and estimation)</li> <li>Use a variety of concrete materials, mental (e.g. commutative and distributive properties) and written strategies to solve problems involving the four operations with fractions, decimals, percentages, ratios and rates.</li> <li>Represent positive and negative fractional and decimal units on a number line.</li> </ul>

## Statistics

Level Outcomes									
1.0.5	Use some of the basic language of chance and probability to describe everyday experiences	2.0.5	Represent, interpret and find patterns in data related to everyday experiences	3.05	Produce a range of data representations to support interpretations	4.05	Apply simple tests to data to explore the concepts of probability and randomness.	5.05	Select samples and appropriate data collection strategies and report their findings using a range of techniques and mathematical concepts.
Contributing Outcomes									
<ul style="list-style-type: none"> <li>▪ Represents simple data sets using objects and pictures;</li> <li>▪ Interprets simple data displays using everyday language.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Gathers and organises data using column and picture graphs;</li> <li>▪ Interprets column and picture graphs;</li> <li>▪ Recognises and describes the element of chance in everyday events;</li> <li>▪ Uses some of the basic language of chance and probability</li> </ul>	<ul style="list-style-type: none"> <li>▪ Gather, interpret, organize and display data using tables and graphs;</li> <li>▪ Describe and compare chance events in social and experimental contexts.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Order the likelihood of simple events on a number line from zero to one;</li> <li>▪ Display and interpret data in graphs with scales of many-to-one correspondence.</li> <li>▪ Interpret and manipulate data to calculate averages (e.g. mean, medium, mode)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Use the four number operations to calculate interest and charges using percentages and decimals;</li> <li>▪ Construct, read and interpret graphs, tables, charts and statistical information;</li> <li>▪ Collect statistical data using appropriate strategies, and analyse data using measures of location and range;</li> <li>▪ Solve probability problems involving simple events.</li> </ul>					

## Measurement

Level Outcomes				
1.06 Use their own language and a variety of informal measurement units to describes and compare time and spatial attributes of everyday experiences	2.0.6 Use the concept of repeated identical units as a basis to estimate, measure and compare time, the spatial attributes of concrete objects and changes	3.06 Use basic time (min, hrs, days), radial units (degrees) and metric units (m, m <sup>2</sup> , litre, kg) to estimate, precisely measure, compare and record time, and the spatial attributes of concrete objects, events and changes	4.06 Use a full range of units to measure time and spatial attributes of objects, events and changes	5.06 Select and apply a variety of calculations to solve problems relating to time, two and three dimensional measurement and rates of change
Contributing Outcomes				
<ul style="list-style-type: none"> <li>▪ Explore and describe attributes of a variety of concrete objects using everyday qualitative language;</li> <li>▪ Compare concrete objects using direct comparison of selected attributes.</li> <li>▪ Explore the use tessellating objects to create two and three dimensional patterns</li> <li>▪ Explore and describe the concept of <i>time</i> in everyday experience using everyday language</li> </ul>	<ul style="list-style-type: none"> <li>▪ Record attributes using informal units</li> <li>▪ Identify common examples of the basic metric units (m, m<sup>2</sup>, litre, kg, °C).</li> <li>▪ Create and represent patterns using tessellating units</li> <li>▪ Identify and describe passage of time using basic units of years, months, weeks, days, hours.</li> <li>▪ Identify and describe angular properties of everyday objects and events</li> </ul>	<ul style="list-style-type: none"> <li>▪ Demonstrate and explain the relationship between the size of the unit and the number of units required to measure the attribute.</li> <li>▪ Observe, record and describe the angular properties of regular geometric shapes using radial units</li> <li>▪ Select the most appropriate for a given measurement task</li> <li>▪ Observe, record and describe the change in attributes of objects and situations over time.</li> <li>▪ Demonstrate and describe relationship between metric units (e.g. 1.0 litre of water weighs 1.0 kg)</li> <li>▪ Use knowledge of attributes of common objects and events to estimate measurements in metric units and time.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Select and use the appropriate smaller formal units (cm, cm<sup>2</sup>, cm<sup>3</sup>, g) to measure attributes</li> <li>▪ Measure, record and describe the angular properties regular geometric shapes using standard radial units</li> <li>▪ Select and use an appropriate device to measure and calculate attributes;</li> <li>▪ Uses concrete examples to derive formulae where appropriate (e.g. area of triangle derived from area of rectangle; volume of prism derived using cubic unit blocks)</li> <li>▪ Record measurements as decimal fractions.</li> <li>▪ Use reference quantities and experience to estimate and predict measurements in metric units and time</li> </ul>	<ul style="list-style-type: none"> <li>▪ Perform calculations for measurement of three dimensional attributes, using standard metric and radial units with formulae where appropriate;</li> <li>▪ Explain how large measurement units are used;</li> <li>▪ Describe and calculate measurements of large quantities;</li> <li>▪ Interpret simple scales to judge distance and area.</li> <li>▪ Estimate, predict, record and calculate rates of change of real-life events (e.g. speed, growth etc)</li> </ul>

## Space

Level Outcomes				
<p><i>1.07 Describe and represent objects and patterns in terms of shape and position using informal language</i></p>	<p><i>2.07 Describe and represent objects and patterns in terms of shape and 2-and 3-dimensional and radial position using basic mathematical language</i></p>	<p><i>3.07 Describe and represent objects and patterns in terms of shape and position using formal mathematical conventions, 2- and 3- dimensional and radial units and language</i></p>	<p><i>4.07 Describe and represent changes in shape and 2-and 3- dimension and radial position of objects and patterns resulting from rotations and transformations using formal mathematical conventions, units and language</i></p>	<p><i>5.07 Analyse, accurately describe and represent a variety of solids and networks and their position.</i></p>
Contributing Outcomes				
<ul style="list-style-type: none"> <li>▪ Manipulate, sort and represent two- and three-dimensional objects and their position, and describe them using everyday language</li> <li>▪ Recognise, describe, create and continue repeating geometric patterns using two and three dimensional objects</li> <li>▪ Use everyday language to describe position</li> <li>▪ Give and follow simple directions.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Sort, describe and represent two- and three-dimensional objects (including cones, cubes, cylinders, spheres and prisms)</li> <li>▪ Recognise two- and three-dimensional objects in pictures and the environment;</li> <li>▪ Create, represent and continue a variety of geometric patterns (e.g. involving symmetry and tessellation)</li> <li>▪ Supply missing elements in a pattern;</li> <li>▪ Represent the position of objects using models and drawings and describes using everyday and basic mathematical language.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Make, compare, describe and name two- and three-dimensional objects and their position using formal language and basic metric (m, m<sup>2</sup>, litre, kg) and radial units;</li> <li>▪ Represent two- and three-dimensional objects in drawings and explain their relationships using basic metric (m, m<sup>2</sup>, litre, kg); and radial units</li> <li>▪ Generate, describe, continue and record different symmetric and tessellating patterns using a variety of strategies;</li> <li>▪ Use simple maps and grids to represent position and to give and follow direction.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Record, analyse, describe and represent changes and shape and position of objects and patterns due to rotation and transformation</li> <li>▪ Identify a variety of two- and three-dimensional objects (including particular prisms, cones and pyramids) on the basis of their 2-and 3- dimensional and radial properties;</li> <li>▪ Visualise, sketch and construct two- and three-dimensional objects given drawings of different views;</li> <li>▪ Record, analyse and describe geometric patterns using tables and words;</li> <li>▪ Use a variety of mapping skills to represent position and to give and follow directions.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Describe and sketch three-dimensional solids including polyhedra, and classify them in terms of their 2-and 3- dimensional and radial properties;</li> <li>▪ Describe and represent a variety of networks;</li> <li>▪ Create, record, analyse and generalise geometric patterns using words and algebraic symbols in a variety of ways.</li> <li>▪ Use compass bearings and a variety of mapping skills to represent position and to give and follow directions</li> <li>▪ Use directed numbers to describe relative position and change of position (e.g. use a number line where zero represents the starting position).</li> </ul>

## Level 1

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
<i>1.01. Use, describe and represent whole numbers to two places</i>	<i>1.5 Use some of the basic language of chance and probability to describe everyday experiences</i>	<i>1.6 Use their own language and a variety of informal measurement units to describes and compare time and spatial attributes of everyday experiences</i>	<i>1.7 Describe and represent objects and patterns in terms of shape and position using informal language</i>
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Recognise numbers in the range 0 to 20 and beyond;</li> <li>▪ Represent numbers in the range 0 to 20 and beyond using objects, words, pictures and then symbols;</li> <li>▪ Count and order numbers in the range 0 to 20 and beyond in correct forward and backward sequence.</li> <li>▪ Develops concept of ten as ten ones or one ten</li> <li>▪ Recognise, describe, create and continue repeating number patterns that increase or decrease</li> </ul>	<ul style="list-style-type: none"> <li>▪ Represents simple data sets using objects and pictures;</li> <li>▪ Interprets simple data displays using everyday language.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Explore and describe attributes of a variety of concrete objects using everyday qualitative language;</li> <li>▪ Compare concrete objects using direct comparison of selected attributes.</li> <li>▪ Explore the use tessellating objects to create two and three dimensional patterns</li> <li>▪ Explore and describe the concept of <i>time</i> in everyday experience using everyday language</li> </ul>	<ul style="list-style-type: none"> <li>▪ Manipulate, sort and represent two- and three-dimensional objects and their position, and describe them using everyday language</li> <li>▪ Recognise, describe, create and continue repeating geometric patterns using two and three dimensional objects</li> <li>▪ Use everyday language to describe position</li> <li>▪ Give and follow simple directions.</li> </ul>
<b>Level Outcomes</b>			
1.02 Add and subtract one and two digit numbers using a variety of strategies			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Use concept of 10's and ones as a countable units</li> <li>▪ Use and describe a variety of mental strategies for basic addition and subtraction using objects, words and pictures in everyday situations;</li> <li>▪ Use a part or whole knowledge of numbers to 10 and 20 to add and subtract</li> <li>▪ Represent simple calculations with diagrams and objects;</li> <li>▪ Record simple calculations using informal methods (e.g. using words and pictures )</li> </ul>			

Number	Statistics	Measurement	Space
Level Outcomes			
1.03 Create and count equal groups in a given collection.			
Contributing Outcomes			
<ul style="list-style-type: none"> <li>▪ Share collections of objects into equal groups</li> <li>▪ Identify and name equal groups according to the number of objects in the group</li> <li>▪ Use equal groups as countable units to find <i>how many</i> equal groups are in a collection (e.g. rhythmic and skip counting strategies)</li> <li>▪ Describe counting in groups using objects, words, pictures and everyday language.</li> </ul>			
Level Outcomes			
1.04 Divide whole objects into parts and identify and describe equal sized parts using fractional names			
Contributing Outcomes			
<ul style="list-style-type: none"> <li>▪ Form equal groups, share and count collections of discrete objects using equal groups as countable units (e.g. share one packet of 10 lollies between 2 people).</li> <li>▪ Share continuous objects into equal parts (e.g. share one cake equally between two people)</li> <li>▪ Describe halves encountered in everyday contexts, as two equal parts of a continuous object;</li> </ul>			

## Level 2

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
2.01 Use, describe and represent whole numbers to at least three places	2.05 Represent, interpret and find patterns in data related to everyday experiences	2.06 Use the concept of repeated identical units as a basis to estimate, measure and compare time, the spatial attributes of concrete objects and changes	2.07 Describe and represent objects and patterns in terms of shape and 2-and 3- dimensional and radial position using basic mathematical language
<b>Contributing Outcome</b>			
<ul style="list-style-type: none"> <li>▪ Count, order, read, group and represent two- and three-digit numbers using objects, words, pictures and then symbols;</li> <li>▪ Count forward and backwards in 100's, 10's and 1's on and off century and decade.</li> <li>▪ Generate, describe and record simple number patterns using a variety of strategies</li> </ul>	<ul style="list-style-type: none"> <li>▪ Gathers and organises data using column and picture graphs;</li> <li>▪ Interprets column and picture graphs;</li> <li>▪ Recognises and describes the element of chance in everyday events;</li> <li>▪ Uses some of the basic language of chance and probability</li> </ul>	<ul style="list-style-type: none"> <li>▪ Record attributes using informal units</li> <li>▪ Identify common examples of the basic metric units (m, m<sup>2</sup>, litre, kg, °C).</li> <li>▪ Create and represent patterns using tessellating units</li> <li>▪ Identify and describe passage of time using basic units of years, months, weeks, days, hours.</li> <li>▪ Identify and describe angular properties of everyday objects and events</li> </ul>	<ul style="list-style-type: none"> <li>▪ Sort, describe and represent two- and three-dimensional objects (including cones, cubes, cylinders, spheres and prisms)</li> <li>▪ Recognise two- and three-dimensional objects in pictures and the environment;</li> <li>▪ Create, represent and continue a variety of geometric patterns (e.g. involving symmetry and tessellation)</li> <li>▪ Supply missing elements in a pattern;</li> <li>▪ Represent the position of objects using models and drawings and describes using everyday and basic mathematical language.</li> </ul>
<b>Level Outcomes</b>			
2.02 Add and subtract three digit numbers using a variety of strategies			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Use a part or whole knowledge of numbers to 100 to add and subtract;</li> <li>▪ Use concept of 100's, 10's and ones as a countable units in a variety of mental strategies to add and subtract;</li> <li>▪ Use objects, words, pictures, symbols and basic formal recording methods for addition and subtraction involving one- and two-digit numbers</li> <li>▪ Create, represent and continue a variety of simple number patterns, supplying missing elements in a pattern;</li> </ul>			

Number	Statistics	Measurement	Space
Level Outcomes			
2.03 Use a range of mental strategies to solve multiplication and division tasks;			
Contributing Outcomes			
<ul style="list-style-type: none"> <li>▪ Use objects, words, pictures, symbols and informal recording methods to represent simple multiplication and division facts;</li> <li>▪ Use a range of mental strategies and concrete materials to coordinate counting of equal groups of visible and concealed objects (e.g. use visual markers to coordinate counting in groups);</li> <li>▪ Apply simple basic multiplication and division facts (e.g. for 1, 2, 4, 5 and 10) to a range of multiplication and division tasks (e.g. dividing whole objects and collections into equal shares; finding how many of a group are in a given collection)</li> </ul>			
Level Outcomes			
2.04 Determine the relationship between the number of fractional parts of a whole or set of objects and the size and names of these fractional parts			
Contributing Outcomes			
<ul style="list-style-type: none"> <li>▪ Explore, create visual models and describe fractional parts (halves and quarters etc) of objects and collections occurring in everyday situations (e.g. share two pizzas equally among four people; share a packet of 12 lollies among four people)</li> <li>▪ Regroup fractional parts into wholes using concrete materials (e.g. combine nine quarters to form two wholes and one quarter)</li> <li>▪ Compare fractions using concrete materials <i>without</i> using standard fraction notation. (e.g. use fraction pieces to show that three quarters is bigger than one half, but smaller than one whole)</li> </ul>			

### Level 3

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
3.01 Use, describe and represent whole numbers to at least four places	3.05 Produce a range of data representations to support interpretations	3.06 Use basic time (min, hrs, days), radial units (degrees) and metric units (m, m <sup>2</sup> , litre, kg) to estimate, precisely measure, compare and record time, and the spatial attributes of concrete objects, events and changes	3.07 Describe and represent objects and patterns in terms of shape and position using formal mathematical conventions, 2-and 3- dimensional and radial units and language
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Count, order, read and record numbers up to four digits using objects, words, pictures and then symbols;</li> <li>▪ Count forward and backwards in 1000's, 100's, 10's and 1's on and off millennium, century and decade.</li> <li>▪ Generate, describe and record number patterns using a variety of strategies;</li> </ul>	<ul style="list-style-type: none"> <li>▪ Gather, interpret, organize and display data using tables and graphs;</li> <li>▪ Describe and compare chance events in social and experimental contexts.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Demonstrate and explain the relationship between the size of the unit and the number of units required to measure the attribute.</li> <li>▪ Observe, record and describe the angular properties of regular geometric shapes using radial units</li> <li>▪ Select the most appropriate for a given measurement task</li> <li>▪ Observe, record and describe the change in attributes of objects and situations over time.</li> <li>▪ Demonstrate and describe relationship between metric units (e.g. 1.0 litre of water weighs 1.0 kg)</li> <li>▪ Use knowledge of attributes of common objects and events to estimate measurements in metric units and time.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Make, compare, describe and name two- and three-dimensional objects and their position using formal language and basic metric (m, m<sup>2</sup>, litre, kg) and radial units;</li> <li>▪ Represent two- and three-dimensional objects in drawings and explain their relationships using basic metric (m, m<sup>2</sup>, litre, kg); and radial units</li> <li>▪ Generate, describe, continue and record different symmetric and tessellating patterns using a variety of strategies;</li> <li>▪ Use simple maps and grids to represent position and to give and follow direction.</li> </ul>

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
3.02 Add and subtract four digit numbers using a variety of strategies			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Use concept of grouping in 100's and 1000's to add and subtract;</li> <li>▪ Use a part or whole knowledge of numbers to 1000 to add and subtract</li> <li>▪ Add and subtract two-, three- and four-digit numbers using a variety of mental and written strategies</li> <li>▪ Complete addition and subtraction number sentences by calculating missing values.</li> <li>▪ Generate, describe, complete and record number patterns using a variety of strategies.</li> </ul>			
<b>Level Outcomes</b>			
3.03 Use mental and written strategies for multiplication and division with two and three digit numbers			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Apply basic multiplication and division facts (e.g. for 3, 6, 9, and 12) and understanding of place value to a range of multiplication and division tasks (e.g. dividing whole objects and collections into equal shares; finding how many of a group are in a given collection).</li> <li>▪ Represent and solve multiplication and division calculations using basic formal algorithms.</li> </ul>			

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
3.04 Represent compare and order fractions and decimal numbers using concrete materials, words and standard notation.			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Create concrete models, compare, order and represent commonly used and equivalent fractions (including thirds, fifths and tenths) by considering the number of fractional parts. (e.g. share three pizzas equally among four people; share two packets of 6 lollies among three people)</li> <li>▪ Demonstrate and explain the relationship between equivalent fractions using concrete materials</li> <li>▪ Represent fractions using concrete models, words and standard fraction notation, and explain the meaning of denominator as the number of fractional parts of a whole or set, and the numerator as the number of fractional parts being considered</li> <li>▪ Create concrete models, compare, order and represent decimal numbers to tenths using a variety of ways (concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings).</li> <li>▪ Add and subtract decimals to one decimal place.</li> <li>▪ Create concrete models, compare, order, represent and interpret everyday percentages;</li> </ul>			

## Level 4

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
4.01 Use, describe and represent whole numbers to at least six places	4.05 Apply simple tests to data to explore the concepts of probability and randomness.	4.06 Use a full range of units to measure time and spatial attributes of objects, events and changes	4.07 Describe and represent changes in shape and 2-and 3-dimension and radial position of objects and patterns resulting from rotations and transformations using formal mathematical conventions, units and language
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Count, Order, read and write numbers of any size using objects, words, pictures and symbols;</li> <li>▪ Record, analyse and describe number patterns that involve one operation using tables and words.</li> <li>▪ Demonstrate sufficient mathematical understandings for most day to day activities</li> </ul>	<ul style="list-style-type: none"> <li>▪ Order the likelihood of simple events on a number line from zero to one;</li> <li>▪ Display and interpret data in graphs with scales of many-to-one correspondence.</li> <li>▪ Interpret and manipulate data to calculate averages (e.g. mean, medium, mode)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Select and use the appropriate smaller formal units (cm, cm<sup>2</sup>, cm<sup>3</sup>, g) to measure attributes</li> <li>▪ Measure, record and describe the angular properties regular geometric shapes using standard radial units</li> <li>▪ Select and use an appropriate device to measure and calculate attributes;</li> <li>▪ Uses concrete examples to derive formulae where appropriate (e.g. area of triangle derived from area of rectangle; volume of prism derived using cubic unit blocks)</li> <li>▪ Record measurements as decimal fractions.</li> <li>▪ Use reference quantities and experience to estimate and predict measurements in metric units and time</li> </ul>	<ul style="list-style-type: none"> <li>▪ Record, analyse, describe and represent changes and shape and position of objects and patterns due to rotation and transformation</li> <li>▪ Identify a variety of two- and three-dimensional objects (including particular prisms, cones and pyramids) on the basis of their 2-and 3- dimensional and radial properties;</li> <li>▪ Visualise, sketch and construct two- and three-dimensional objects given drawings of different views;</li> <li>▪ Record, analyse and describe geometric patterns using tables and words;</li> <li>▪ Use a variety of mapping skills to represent position and to give and follow directions.</li> </ul>
<b>Level Outcomes</b>			
4.02 Add and subtract six digit numbers using a variety of strategies			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Construct, verify and complete number sentences involving the addition and subtraction operations with a variety of numbers;</li> <li>▪ Add and subtract counting numbers of any size using appropriate strategies;</li> <li>▪ Apply formal algorithms to addition and subtraction tasks.</li> </ul>			

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
4.03 Select and apply appropriate strategies to solve problems involving multiplication and division			
<b>Contributing Outcomes</b>			
4.03 <ul style="list-style-type: none"> <li>▪ Select and apply a appropriate formal algorithms to a wide range of problems requiring mathematical calculations</li> <li>▪ Represent multiplication and division tasks as formal algorithms.</li> <li>▪ Construct, verify and complete number sentences involving the multiplication and division with a variety of numbers.</li> </ul>			
<b>Level Outcomes</b>			
4.04 Represent, compare, order and perform calculations with fractions, decimals, ratios and percentages using concrete materials, words and standard notation..			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Represent, compare and order fractional amounts with like and unlike denominators including proper and improper fractions and mixed numbers in a variety of ways and standard fractional notation.</li> <li>▪ Demonstrate and explain the concept of equivalent fractions using concrete materials and drawings.</li> <li>▪ Round decimal numbers to nearest tenth in problems arising from real life situations.</li> <li>▪ Create concrete models, compare, order and represent decimal numbers to thousandths using a variety of ways (concrete materials: paper strips divided into tenths and base ten materials, number lines, drawings)</li> <li>▪ Estimate quantities using benchmarks of 10%, 25%, 50%, 75%, and 100% (e.g. the container is 75% full; approx. 50% of students bus to school)</li> <li>▪ Compare, order and calculate with decimals, simple fractions and percentages, ratios and rates.</li> </ul>			

## Level 5

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
5.01 Use and apply place value to represent very large and very small numbers and directed numbers	5.05 Select samples and appropriate data collection strategies and report their findings using a range of techniques and mathematical concepts.	5.06 Select and apply a variety of calculations to solve problems relating to time, two and three dimensional measurement and rates of change	5.07 Analyse, accurately describe and represent a variety of solids and networks and their position.
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Read and write very large and small numbers;</li> <li>▪ Use very large and small numbers in calculations with formal algorithms.</li> <li>▪ Read and write one and two digit directed numbers</li> </ul>	<ul style="list-style-type: none"> <li>▪ Use the four number operations to calculate interest and charges using percentages and decimals;</li> <li>▪ Construct, read and interpret graphs, tables, charts and statistical information;</li> <li>▪ Collect statistical data using appropriate strategies, and analyse data using measures of location and range;</li> <li>▪ Solve probability problems involving simple events.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Perform calculations for measurement of three dimensional attributes, using standard metric and radial units with formulae where appropriate;</li> <li>▪ Explain how large measurement units are used;</li> <li>▪ Describe and calculate measurements of large quantities;</li> <li>▪ Interpret simple scales to judge distance and area.</li> <li>▪ Estimate, predict, record and calculate rates of change of real-life events (e.g. speed, growth etc)</li> </ul>	<ul style="list-style-type: none"> <li>▪ Describe and sketch three-dimensional solids including polyhedra, and classify them in terms of their 2-and 3-dimensional and radial properties;</li> <li>▪ Describe and represent a variety of networks;</li> <li>▪ Create, record, analyse and generalise geometric patterns using words and algebraic symbols in a variety of ways.</li> <li>▪ Use compass bearings and a variety of mapping skills to represent position and to give and follow directions</li> <li>▪ Use directed numbers to describe relative position and change of position (e.g. use a number line where zero represents the starting position).</li> </ul>
<b>Level Outcomes</b>			
5.02 Perform operations with numbers to one million and understand decimals to one millionth.			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Recognises the properties of special groups of whole numbers and apply a range of strategies to aid computation;</li> <li>▪ Compares, orders and calculates with integers.</li> <li>▪ Add and subtract one and two digit directed numbers on a number line</li> </ul>			

Number	Statistics	Measurement	Space
<b>Level Outcomes</b>			
5.03 Explain the relationships between numbers using simple algebra and coordinates			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Describe the relationships between numbers and number patterns using words and simple algebra and coordinates in a variety of ways;</li> <li>▪ Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions;</li> <li>▪ Uses algebraic techniques to solve linear equations and simple inequalities.</li> <li>▪ Multiply and divide one and two digit directed numbers on a number line</li> </ul>			
<b>Level Outcomes</b>			
5.04 Use a variety of mental and written strategies to solve problems involving all four operations with fractions, decimals, ratios and percentages.			
<b>Contributing Outcomes</b>			
<ul style="list-style-type: none"> <li>▪ Demonstrate a single number sense of fractions;</li> <li>▪ Order fractions by using multiplicative structure to create equivalences and estimate location;</li> <li>▪ Add, subtract, multiply and divide whole numbers by simple fractions and by decimal numbers to hundredths using concrete materials (e.g. divide 3 by <math>\frac{1}{2}</math> using fraction strips; divide 4 by 0.8 using base ten materials and estimation)</li> <li>▪ Use a variety of concrete materials, mental (e.g. commutative and distributive properties) and written strategies to solve problems involving the four operations with fractions, decimals, percentages, ratios and rates.</li> <li>▪ Represent positive and negative fractional and decimal units on a number line.</li> </ul>			

## Teaching And Learning In Mathematics

The mathematics curriculum encourages teachers to develop their use of reflective teaching practices in order to promote improved learning outcomes for their students.

Like most of the curriculum, the teaching of mathematics has changed considerably over the past 25 or so years. The days of the teacher providing repetitive drill and practice have given way to classrooms where students learn through investigation, the manipulation of concrete materials, interaction with technology and the solving of real world problems.

Of course, time for practice and reinforcement is still a vital part of the mathematics program and the ability to recall basic facts remains an essential skill. Today's classrooms, however, strive for deeper understandings, greater enthusiasm and a clear sense of relevance. There is a clear focus for developing efficient mental strategies to solve numerical problems. Formal written algorithms are introduced after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two-digit and three-digit numbers.

The following section deals with a number of important issues related to the planning, implementation and assessment of mathematics learning.

### *Some Important Classroom Issues*

Because parents and the community attach a great deal of importance to mathematics learning, a number of issues arise from time to time which cause widespread discussion. Schools and individual teachers need to be clear about their beliefs regarding these issues so that classroom practice is consistent and clearly contributes to high standards of mathematical learning. The following five issues are discussed below:

- Problem Solving
- Mental Calculation
- Using Calculators
- Forming Student Groups
- Language and Symbols
- Environments that provide Optimal Conditions for Learning.

### **Problem Solving**

Problem solving has become the most central element in modern mathematics education. Indeed, the ability to analyse and solve problems is one of the IEA's five key outcomes for all students. It provides for the development and demonstration of integrated mathematical understandings in practical and relevant contexts.

There is sometimes confusion over precisely what constitutes a problem. The key to the definition of a problem is that it presents some form of challenge in its solution. There must be no immediately obvious solution. Consequently, a question which can be legitimately labelled a problem for students at Level 2 may no longer be a problem when presented to Level 5 students. Problems which are linked together around a broader topic can be referred to as investigations. Within this context the skill of problem finding becomes as important as problem solving.

Problem solving is both a tool for learning and an end in itself. Teachers need to help students develop a range of strategies which can be applied to problems. At the same time, the opportunity to solve a problem often leads to new understandings about a range of mathematical

ideas. Problem solving provides for high levels of student motivation and improves the learning of basic mathematical concepts.

Problems should not be purely mathematical in content. They can provide a wide range of opportunities for the integration of other curriculum areas, and the skills and strategies developed can be transferred to other areas.

### **Mental Calculation**

There is little doubt that one of the most valuable legacies of a sound education in mathematics is the ability to perform calculations mentally. Rarely does a day pass when there isn't some need to solve a simple mathematical question without the help of pencil and paper. While many of these questions are related to numbers (especially money), others deal with measurement, space or statistics.

McIntosh (1990) reported the results of a survey of teachers and students who were asked to estimate what proportion of the time devoted to computation in primary schools is concerned with (a) written computation, (b) calculator use and (c) mental computation. He found the results to be (a) 90%, (b) <5%, (c) 5%. When the same groups were asked to estimate the proportion of computations done by the majority of people in the normal course of their lives using each of the three methods he found the pattern to be: (a) 10%, (b) 15%, and (c) 75%. These figures should give cause for reflection to all teachers.

One element of successful mental calculations centres around the learning of basic number facts which can be applied in a range of situations. If, for example, students learn and can readily recall that  $6 \times 3 = 18$  they have a valuable piece of information if they need to determine  $16 \times 3$  or  $36 \times 6$ .

While the learning of these basic number facts is important, it is of little value if it is not accompanied by the more important element of mental strategies. In real life we are faced with a wide variety of problems to solve. Being able to use a range of efficient mental strategies provides a facility to solve a variety of numerical problems. To calculate  $36 \times 6$  mentally, some people will copy a pencil and paper model:  $6 \times 6 = 36$ , remember the 6 and carry the three,  $6 \times 3 = 18$ , add the 3 to make 21, remember this is  $21 \times 10 = 210$ , then add the 6 remembered from the original calculation to make 216.

While this approach gives the correct answer it may not be the most useful. We could, for example, work in the opposite direction than we might with paper and say  $6 \times 30 = 180$ , then add  $6 \times 6 = 36$  to arrive at 216. Different people will be more comfortable with different strategies, but all those who are quick and accurate with mental computation will have a good range of strategies at their disposal.

The question of rote learning of tables and facts is related to this issue. In practice, however, this should not be an issue at all. Students should learn their basic number facts through regular and thorough exposure to the numbers involved. They should discover, for example, that  $6 \times 3 = 18$  through studying the number 18 and the number 3, through doubling 9, and so forth. Students who become comfortable with numbers in this way will know their number facts just as surely as if they were drilled in by rote. As a general principle, formal written algorithms are introduced only after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two-digit and three-digit numbers.

## Using Calculators

The use of calculators in mathematics classrooms has been a contentious issue for some years. While calculators are now commonplace and unquestionably have a role to play in mathematics education, schools are wise to develop a clear rationale for their use.

The main objection to calculators centres around the argument that if students use calculators they will never learn to cope without them, and will fail to develop a clear understanding of the operations they are performing. It is certainly alarming to find shop assistants who reach for a calculator to add 90t and 20t or calculate a five percent discount on a one Kina purchase. Even more alarming is the insistence of many shop owners that their staff use calculators for even the simplest calculations.

In practice, however, most people use calculators when they want to perform more complex operations quickly and with accuracy. The use of the calculator has become a legitimate life skill, and schools should ensure that students have this skill as well as the ability to perform the same calculations manually. Naturally students must be able to identify the ‘reasonableness’ of their results to confirm the accuracy of calculations and processing.

In simple terms, students should be allowed and even encouraged, to use calculators to perform calculations which they are clearly capable of doing manually. If the student can determine which processes are necessary to find a solution to a mathematical problem then they have demonstrated mathematical understanding at a more sophisticated level than through the performance of manual calculations.

Of course, some mathematical calculations in the higher levels are so complex that it would be nearly impossible to complete them without technological assistance. One of the great advantages of the calculator is that it allows students to study aspects of mathematics which were previously too calculation intense to be practical.

## Language and Symbols

Language plays an essential part in the formulation and expression of mathematical ideas and students should be encouraged to see it as a tool for their mathematical learning. The use of the child’s existing language to explain ideas and understandings needs to be encouraged. This language acts as a bridge to the more formal mathematical language. Children have many mathematical ideas that they can deal with confidently if allowed to express them in their own language. As they gain more experience with the concepts and the language involved, students will make closer approximations towards mathematical language.

Historically, the writing that has occurred in mathematics has taken the form of charts, tables, graphs or symbols. The role of writing as a way of exploring mathematical ideas and concepts, clarifying thoughts, expressing understandings and recording feelings and attitudes has been largely unexplored or ignored.

Mathematical concepts do not develop apart from the language used to express them. Students are likely to develop these ideas more readily if they have the opportunity to express their thoughts in many ways. By writing, students are forced to clarify their ideas, can keep a record of their thoughts for future reference and are not reliant on immediate assistance or feedback. There are many different types of writing that can be used in mathematics. Teachers need to be aware of taking opportunities to develop various forms of writing in a meaningful context.

Reading skills in mathematics should be developed alongside other reading skills so that children can understand the explanations and instructions occurring in the mathematical books they use. They should learn to be selective and to edit the information with which they are presented.

Teachers need to be aware of the great variety of language which is used in connection with many of the mathematical operations which children will meet so that they can assist their pupils to become familiar with the different ways in which the same mathematical idea can be expressed. For example, the instructions “add 5 and 3”, “add 3 to 5”, “find the sum of 5 and 3”, “find the number which is 3 more than 5” all require the same mathematical operation to be carried out. Children need to be able to interpret these apparently different instructions and to attach them all to the symbolic form  $5+3$ .

Symbolic expressions are used in mathematics to express mathematical thought and processes precisely and concisely. Mathematical symbols are a form of representation which must be preceded by experiences which allow the mathematics learner to internalise the meaning behind the symbol.

The manipulation of concrete materials, the use of everyday language followed by the transition to mathematical language and the use of pictorial representations must all take place before the introduction of a mathematical symbols. Sufficient activities involving materials and discussion of concepts and ideas before the introduction of the symbol will ensure little difficulty for the student in accepting the meaning behind the symbol.

It is important to remember that the symbols do not have anything inherent in them which makes their meaning clear.

### **Materials and Equipment**

Because mathematics is concerned with investigating aspects of the real world, and is best learnt through practical concrete experiences, there is virtually no limit to the items which can be used as resources in the mathematics classroom. The following list may be helpful in establishing a well resourced mathematics learning environment.

flat shapes	paint	timetables
3-D shapes	play dough	drawings
unifix	Lego	measuring tapes
counters	collage materials	confetti
icy-pole sticks	plastic containers	streamers
balance scales	kitchen scales	bathroom scales
cubes	MAB (base 10 blocks)	constructing straws
dice sets	grid paper	string
centicubes	books	sand
matchsticks	newspapers	water
pasta	cards	birthday chart
plastic people	mirrors	hoops
daisy links	thermometers	ropes
straws	stamps and stamp pads	games

Cuisenaire	computer	bingo
bottle tops	number board	weights
rulers	abacus	rocks
compass	calculators	trundle wheel
shells	protractors	geo boards
beans	egg cartons	toy cars
beads	money	graphs
attribute blocks	coloured paper	numeral expanders
place value tabs	place value charts	pattern blocks
fraction blocks	Johnson number lines	metre rules
polygons	geoshapes	isometrics dot paper
Guinness Book of Records	lego	wooden blocks
containers and funnels	analogue and digital clocks	